A STOCHASTIC REPAIRABLE ITEM INVENTORY MODEL

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THESIS

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INVENTORY MODEL

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ABSTRACT

A model is constructed of a repairable item inventory system where on-hand stocks are resupplied both by repair and procurement. The basic properties of the model are analyzed. A Poisson failure process is assumed, and steady state probabilities are found for operation of the model under <R,Q> and <S-1,S> inventory policies. Further, a compound Poisson failure process under an <S-1,S> inventory policy is also analyzed. Measures of supply performance, such as expected number of backorders per unit time, are derived for the system under the various policies.

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TABLE OF SYMBOLS AND ABBREVIATIONS

C_1	number of items in repair, $X(t)$, at $t=0$
C_2	number of items on order, $Z(t)$, at $t=0$
C_3	net inventory, $Y(t)$, at $t=0$
D	delay between item failure and establishment of a purchase order for a replacement; a random variable.
$D_1(t)$	number of items repaired and returned to stock in the interval $(0,t]$; a random variable.
$D_2(t)$	number of items ordered and received in the interval $(0,t]$; a random variable.
f_L	probability density function of procurement lead time.
f_R	probability density function of repair time.
f_U	probability density function of batch size.
F_L	probability distribution function of procurement lead time.
F_R	probability distribution function of repair time.
$H(t)$	the number of items on hand (in stock minus backorders) at any time t; a random variable.
I	time required to inspect an item; a random variable.
ICP	Inventory Control Point
$I(t)$	inventory position, i.e., on hand + in repair + on order, at any time t; a random variable.
L	mean procurement lead time.
L_o	time between placement of an order and receipt of material into stock; a random variable.
$N_1(t)$	number of failures entering repair during an interval of length t.
$N_2(t)$	number of failures which are lost to the system during an interval of length t.
$N_2'(t)$	number of failures placed on order during an interval of length t.
p	probability an item which has failed is repairable

p_1 probability a failed item is returned to the repair depot.

p_2 probability an item which arrives at the repair depot can actually be repaired.

q probability an item which has failed is lost to the system.

Q the amount of each order.

r probability a batch of failures is repairable.

R reorder level.

R_A mean repair time.

R_0 time required to repair an item; a random variable.

$\langle R,Q \rangle$ type of inventory policy whereby an order is placed for Q items whenever inventory position first reaches $x \leq R$.

$\langle s,S \rangle$ type of inventory policy whereby an order is placed for S-x items whenever inventory position first reaches $x \leq s$.

T_1 in-transit delay time, customer to inspection; a random variable.

T_2 in-transit delay time, repair depot to stock depot; a random variable.

U number of failures per batch; a random variable.

U_i number of failures in the i^{th} batch; a random variable.

$V(t)$ number of batches of failures in an interval $(s,s+t]$.

$V_1(t)$ number of batches of repairable failures in an interval $(s,s+t]$.

$V_2(t)$ number of batches which are lost to the system in an interval $(s,s+t]$.

$W(t)$ number of items which fail in an interval $(s,s+t]$.

$X(t)$ number of items in repair at any time t.

$Y(t)$ net inventory (on hand and in repair) at any time t.

$Z(t)$ number of items on order at any time t.

λ rate of a Poisson process.

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I. INTRODUCTION

Although inventory theory was one of the first areas to which operations research analysts turned their attention, most of the work has been concerned with the control of consumable items, i.e. items that are consumed in use. Until fairly recently the control of repairable items did not excite a great deal of effort. However, it has now been realized that, at least in military applications, repairable or recoverable items are a major concern of inventory managers, both materially and financially. Repairables often account for a major portion of the total dollar investment even though the number of items may be small compared to the number of consumable items. Further, because repairable items are generally complex assemblies, they often require special storage, handling, packaging, and shipment methods.

One of the major difficulties of applying inventory theory to consumable items is the extreme variability of item parameters, such as demand, procurement lead time, sources of supply, etc. Further, the very fact of very large numbers of items complicates an already difficult task. Repairable items, on the other hand, have had a tendency to minimize some of these problems. The total number of items is much smaller and it seems somewhat easier to determine accurate parameters. For instance, a large number of repairable items may be classified electronic sub-components. Often, these can be further grouped and then treated as a group. The sources of supply and the sources of repair are generally completely known and limited in number. Repair lead times and procurement lead times may be often determined quite accurately with limited variances. Demand is generally more stable in the sense that if

a given repairable item is a sub-component of a larger unit, it is generally easy to find out how many larger units are in use, and this at least puts an upper bound on demand.

On the other hand, the control of inventories of repairable items is complicated by the fact of two "sources of supply", i.e., repair or procurement. The purpose of this thesis is to analyze a model of a repairable item inventory system under some common inventory policies assuming Poisson and Compound Poisson demand processes. The question of optimizing the system under the various policies is not taken up, since inventory managers and other military decision-makers have not yet agreed as to the appropriate optimizing criteria.

II. A MODEL OF A REPAIRABLE ITEM INVENTORY SYSTEM

The model used in this paper is a fairly close approximation of the actual repairable item inventory system in use in the Navy today. Almost every item stocked by Naval vessels and installations is assigned for technical and management control to some Inventory Control Point (ICP). Based on economic and technical considerations, the ICP will designate certain items as "repairable." These items are carried in stock, at various stock points or depots scattered throughout the Navy Supply System. Although for some items the ICP does not know how much of an item a certain stock depot has, repairable items are almost always "transaction reporting" items. This means that all issues, receipts, losses, etc. of that item at the Stock Depots are reported daily to the ICP, which closely manages the item at a central level.

Since items are often designated as repairable due to high procurement cost or critically low system stocks, the ICP is vitally interested in regaining all items which have failed. To facilitate this return of failed items, ships or stations requesting issue of repairable items are required to certify that a like number of failed items "has been or will be turned in." However, due to the ship's operating position, distance from stock points or repair depots, human procrastination, etc. there may be some delay in failed items arriving at the repair shop. Some items may be lost in transit or may never be turned in. Those that do arrive at the repair depot are inspected and some are found not-fit-for-repair and are discarded for salvage value or cannabalized for usable sub-parts. Those that can be repaired are eventually returned to the stock depots for reissue. Meanwhile the ICP, observing some items being lost to the

system, may decide to procure some new items to bring the total system stock back to some desired level. The system just described is shown in Figure 1.

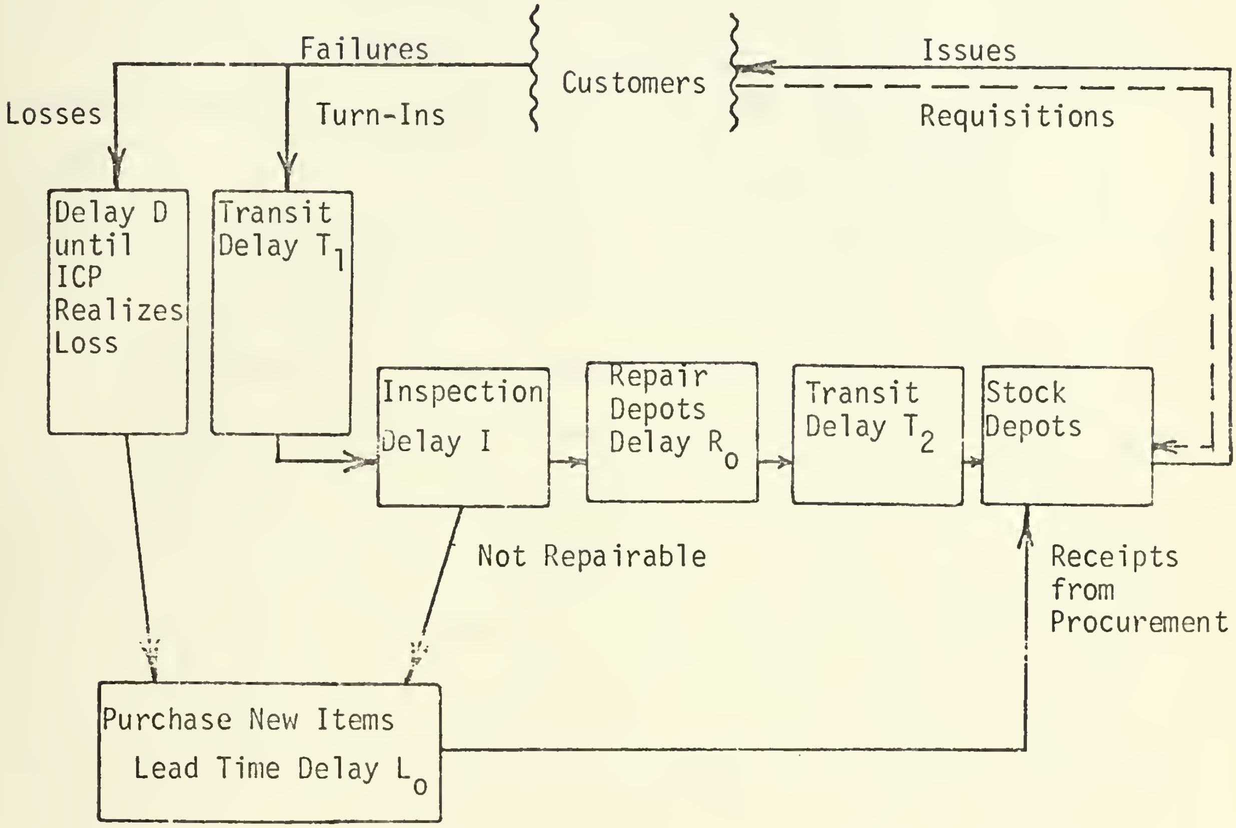

Figure 1. Actual Repairable Item Inventory System

It should be noted that in actuality there usually will be several stock depots and probably at least two repair depots.

However, this system can be considerably simplified with little or no loss of generality if it is considered from the point of view of the ICP. Typically, an ICP will know only when items are issued, when items are received into stock from repair or procurement, and when failures are turned in to the repair depots. From this information, the total number in system stock or in repair at any time can be calculated easily. The ICP controls the procurement of new items and thus knows the number on

order at any time. From these remarks, it can be seen that the various delays in the system, as seen from the ICP, are merely parts of either delay in repair or delay while on order. Therefore, it will be assumed that items which are repairable undergo a "repair time" which is the time from issue of the replacement item out of stock to receipt of the corresponding repaired item back into stock. Repair time in Figure 1 is therefore $T_1 + I + R_0 + T_2$. Likewise, "procurement lead time" is the time between issue and return to stock of a replacement for items which are lost or not repairable and thus must be purchased.

The basic quality of failed items is whether they are repairable or must be replaced. Assume failed items are turned into the repair depot with probability p_1 and of those an item is repairable with probability p_2, then failed items end up in repair with probability p_1p_2 and end up in new procurement with probability $(1 - p_1) + p_1(1 - p_2) = 1 - p_1p_2$. Now, to simplify the model, assume that T_1, T_2, I, and D are equal to zero, or equivalently that the ICP has complete, instantaneous knowledge of the location and condition of all items, and can develop repair times and procurement lead times as given above.

Let mean repair time be denoted by R_A. Mean procurement lead time will be denoted by L. Assume that R_A has a probability density function f_R and probability distribution function F_R. Likewise L has density f_L and distribution F_L. Let $p_1p_2 = p$ and $1 - p_1p_2 = q$. Then the system described above can now be modeled as shown in Figure 2. Assume that failure of any item is independent of the failure or non-failure of other items. Further assume that the number of customer requisitions in any period of time is a Poisson random variable with mean rate λ. The number of items requested per requisition may be one each, in which case the

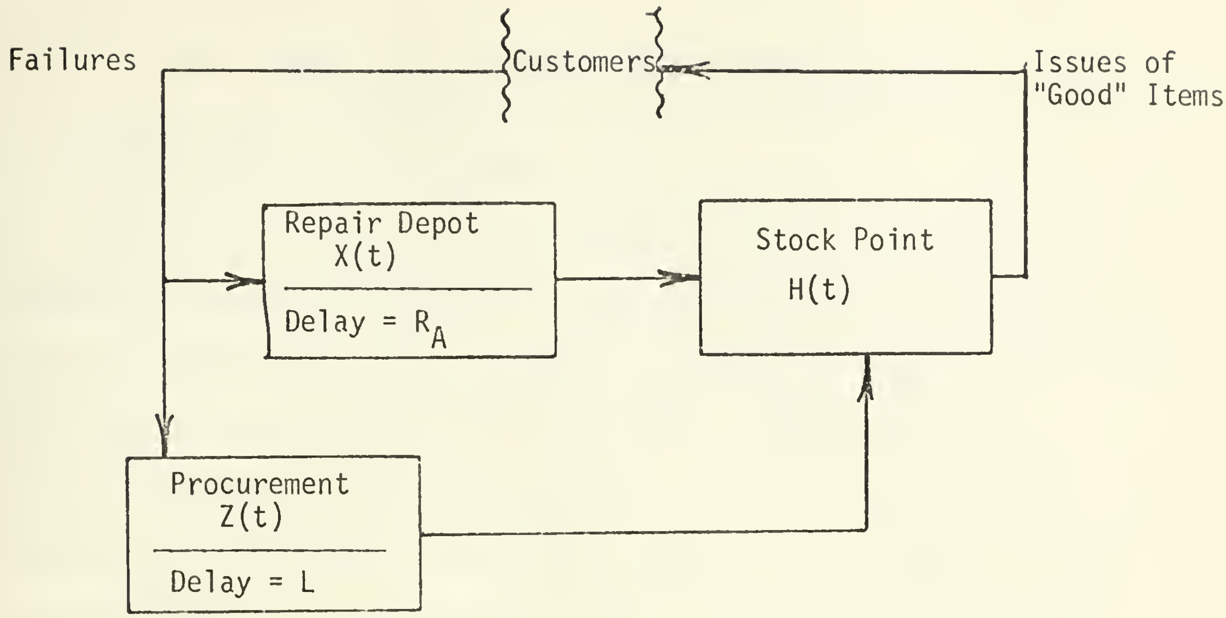

Figure 2. Model of the Repairable Item Inventory System

process is a simple Poisson process. If more than one item can be requested, the process is a compound Poisson process. The number of items in repair at any time t is X(t). The number of items on order at any time t is Z(t). The number on hand, which is the number in stock minus the number of backorders, at any time t is denoted H(t). The net inventory of the system, defined as the number in stock plus in repair minus backorders at any time t is Y(t) = X(t) + H(t).

III. ANALYSIS OF THE MODEL UNDER POISSON DEMAND

A. GENERAL ANALYSIS OF THE MODEL

The number of failures in any interval $(s,s+t]$ is assumed to be Poisson distributed with mean λt. From the assumptions in Section II, it is seen that every failure results in either an additional item undergoing repair or an additional item on order. Assuming that the classification of failures are mutually independent and independent of the process, it has been shown [Parzen 1962] that the number of items entering repair in the interval $(s,s+t]$ is Poisson with mean rate λp. Likewise, the number of failures which will be lost to the system in the interval $(s,s+t]$ is Poisson with mean rate λq. Further, the conditional distribution of the number of items entering repair in an interval $(s,s+t]$ given that n failures have occured in that interval is binomial with parameters n and p.

Now it can be shown that the number of failures entering repair in a given interval is independent of the number of failures which are lost to the system. Let $N_1(t)$ be the number of failures entering repair during an interval of length t and $N_2(t)$ be the number of failures which are lost to the system during the same interval. Then the joint probability mass function for both types of failures is:

$$P(N_1(t) = x,\ N_2(t) = n-x)$$

$$= P(N_1(t) = x,\ N(t) = n)$$

$$= P(N_1(t) = x \mid N(t) = n)\ P(N(t) = n)$$

$$= \binom{n}{x} p^x (1-p)^{n-x} \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

$$= \frac{n!}{x!(n-x)!} p^x(1-p)^{n-x} \frac{(\lambda t)^x(\lambda t)^{n-x}e^{-p\lambda t}e^{-(1-p)\lambda t}}{n!}$$

$$= \frac{p^x(\lambda t)^x e^{-p\lambda t}}{x!} \cdot \frac{(1-p)^{n-x}(\lambda t)^{n-x}e^{-(1-p)\lambda t}}{(n-x)!}$$

$$= \frac{(p\lambda t)^x e^{-p\lambda t}}{x!} \frac{(q\lambda t)^{n-x}e^{-q\lambda t}}{(n-x)!}$$

$$= P(N_1(t)=x)\ P(N_2(t)=n-x)$$

Thus $N_1(t)$ and $N_2(t)$ are independent.

At this point, before further analysis can be done, the processes of repair and procurement must be given some sort of formal definition. The number of items in the repair depot can be viewed as the number of busy servers in a queue, specifically an infinite server queue. For items being repaired, the idea of an infinite server queue is not completely satisfactory. Certainly any repair depot set up to handle a continuing stream of failures will probably have the ability to work on more than one item at a time. However, depending on the Poisson failure process and the repair time distribution, the probability of large numbers of items entering or being in repair at any time may not be insignificant. Nevertheless, the assumption will be made that the repair process can be considered an infinite server queue, i.e. the repair shop immediately starts work on all items as soon as they are received.

The other specification necessary for analysis is the form of the reorder policy. The inventory systems considered in this paper are all continuous review systems, that is, all transactions are recorded immediately, and thus, the inventory manager knows the exact value of stock

on hand, on order, or in repair at any time. A common continuous review policy is the <R,Q> policy, which specifies that a constant amount Q be ordered whenever the inventory position reaches R. The inventory position is defined as on hand plus on order plus in repair minus backorders. Under this policy the procurement process may be thought of as a queue where the number of busy servers at a time t represents the number of orders of size Q that are outstanding at that time. Another frequently used policy is the <s,S> policy, meaning order up to S whenever the inventory position falls below s. In the case where s = S-1, an order is placed everytime a failure cannot be repaired. Under the <s,S> policy, the procurement process is also comparable to an infinite server queue where the number of busy servers at a time t is the number of orders outstanding at that time. When items are demanded one at a time, the <R,Q> policy is the same as an <s,S> policy.

Now let $X(t)$ be the number of items undergoing repair at time t and $Z(t)$ be the number of items on order at time t. Then

$$X(t) = N_1(t) + C_1 - D_1(t)$$

$$Z(t) = N_2'(t) + C_2 - D_2(t)$$

where C_1 is $X(0)$, C_2 is $Z(0)$, $D_1(t)$ is the number of items repaired and returned to stock in the interval $(0,t]$, $D_2(t)$ is the number of items ordered and received in the interval $(0,t]$, and $N_2'(t)$ is the number of items lost to the system which have been placed on order by the time t. $N_2'(t)$ is seen to be related to $N_2(t)$ by:

$$N_2'(t) = Q \left[\!\left[\frac{N_2(t)}{Q} \right]\!\right]^-$$

where $[\![n]\!]^-$ is the largest integer less than n. If items are reordered every time one is lost to the system, i.e. Q=1, then $N_2'(t) = N_2(t)$.

Otherwise $N_2'(t) \leq N_2(t)$. Now both X(t) and Z(t) can be likened to the number of busy servers in a queue. $D_1(t)$ depends only on $N_1(t)$, the repair queue discipline, and repair time distribution. $N_2'(t)$ and $D_2(t)$ depend only on $N_2(t)$, the order queue discipline and procurement lead time distribution. Assuming independence between the two queue operations, it is seen that X(t) and Z(t) are independent.

Let H(t) be the number of items in stock ready for issue at time t, where H(t) is negative if backorders exist. The net inventory, Y(t) = X(t) + H(t), is the number of items in stock plus in repair minus backorders. It is important for further analysis, that the independence of X(t) and Y(t) be shown, since H(t) will be obtained from X(t) and Y(t). This independence has been shown by Richards [Ref. 5] and it is instructive to repeat the proof here:

"...examine the manner in which transitions are made in the two stochastic processes, $\{X(t), t \geq 0\}$ and $\{Y(t), t \geq 0\}$.

Whenever a type-1 failure [into repair] occurs, H(t) decreases by one, but X(t) also increases by one for a net change of zero in Y(t). On the other hand, H(t) decreases by one while X(t) remains constant when a type-2 failure [procurement] occurs; thus a type-2 failure causes a net decrease of one in Y(t) but does not affect X(t).

The arrival of a procurement increases Y(t) by the amount Q, while the repair level is unaffected. Conversely, X(t) decreases while H(t) increases by the like amount whenever an item leaves repair. Thus, repairs decrease X(t) but leave Y(t) unchanged."

Then it can be seen that

$$X(t) = C_1 + N_1(t) - D_1(t)$$

$$Y(t) = C_3 - N_2'(t) + D_2(t)$$

where $C_3 = Y(0)$. As previously shown $D_1(t)$ depends only on $N_1(t)$ and not on $N_2(t)$. Conversely, $N_2'(t)$ and $D_2(t)$ are functions of $N_2(t)$ but not $N_1(t)$. Thus $X(t)$ is a function only of $N_1(t)$ while $Y(t)$ is a function only of $N_2(t)$. Since $N_1(t)$ and $N_2(t)$ have been shown to be independent, then $X(t)$ and $Y(t)$ are independent.

B. AN <R,Q> POLICY WITH CONSTANT PROCUREMENT LEAD TIME

Consider first the operation of the model under an <R,Q> policy, where the inventory position $I(t) = X(t) + H(t) + Z(t) = Y(t) + Z(t)$. An order is placed for Q units whenever $I(t)$ falls to R. Assume that all delay times other than repair time and procurement lead time are zero. Further assume that procurement lead time is a constant, L. Then the "net demand" placed on the system stock, $Y(t)$, i.e. the number of items which must be replaced through procurement is a Poisson process of rate q. Thus the model from this point of view is merely an inventory system operating under an <R,Q> policy experiencing Poisson demands at rate q. This model has been solved by Hadley and Whitin [Ref. 2]. Translating their notation into that used in this paper, the following are given:

$$P[Y(t)=y] = \frac{1}{Q}\left[\overline{F}(R+1-y;\lambda qL) - \overline{F}(R+Q+1-y;\lambda qL)\right] \qquad -\infty \le y \le R+1$$

$$= \frac{1}{Q}\left[1 - \overline{F}(R+Q+1-y;\lambda qL)\right] \qquad R+1 \le y \le R+Q$$

$$\text{where} \quad \overline{F}(a;r) = 1 - \sum_{i=a}^{\infty} \frac{(r)^i e^{-r}}{i!}$$

i.e, the complementary cumulative of a Poisson distribution of rate r.

Now

$$Y(t) = X(t) + H(t)$$

thus

$$H(t) = Y(t) - X(t)$$

$$P[H(t)=h] = P[Y(t)-X(t)=h]$$

$$= \sum_{i=0}^{\infty} P[Y(t)=i+h, X(t)=i]$$

Since it has been shown that Y(t) and X(t) are independent

$$P[H(t)=h] = \sum_{i=0}^{\infty} P[Y(t)=i+h]\, P[X(t)=i]$$

Note than an order is placed for Q units whenever the inventory position falls to R. Thus I(t) varies between R + 1 and R + Q. But

$$R + 1 \leq I(t) \leq R + Q$$

implies

$$R + 1 \leq Z(t) + Y(t) \leq R + Q$$

Z(t), the number of items on order, is always non-negative, i.e. $Z(t) \geq 0$

Thus

$$Z(t) + Y(t) \leq R + Q$$

implies

$$Y(t) \leq R + Q - Z(t) \leq R + Q$$

or $\quad \text{Max } Y(t) = R + Q \quad \text{when } Z(t) = 0$

Therefore $\quad P[Y(t)=y] = 0 \quad \text{for } y > R + Q$

means $\quad P[H(t)=h] = \sum_{i=0}^{R+Q-h} P[Y(t)=i+h]\, P[X(t)=i]$

The probability that Y(t) = y has already been developed. Now P[X(t)=x] is merely an application of Palm's queuing theorem which states that for

an infinite server queue, with Poisson arrivals of rate λ and service times distributed $F(t)$ with mean M, the steady state probabilities of x busy servers are Poisson with rate λM, i.e. in this case,

$$P[X(t)=x] = \frac{(\lambda pR_A)^x e^{-\lambda pR_A}}{x!}$$

Since mean repair time is taken to be R_A, and of course

$$E[X(t)] = \lambda pR_A$$

Then

$$P[H(t)=h] = \sum_{i=0}^{R+Q-h} P[Y(t)=i+h]\, P[X(t)=i]$$

$$= \sum_{i=0}^{R+Q-h} \frac{1}{Q}\,[\bar{F}(R+1-i-h;\lambda qL)-\bar{F}(R+Q+1-i-h;\lambda qL)]\; p(i;\lambda pR_A) \quad \text{for } i+h < R+1$$

$$= \sum_{i=0}^{R+Q-h} \frac{1}{Q}\,[1 - \bar{F}(R+Q+1-i-h;\lambda qL)]\; p(i;\lambda pR_A) \quad \text{for } R+1 \le i+h \le R+Q$$

where $p(a;r) = \dfrac{(r)^a e^{-r}}{a!}$

Simplifying

$$P[H(t)=h] = \frac{1}{Q} \sum_{i=0}^{R+Q-h} [1 - F(R-h-i;\lambda qL) - 1 + F(R+Q-h-i;\lambda qL)] \cdot p(i;\lambda pR_A)$$

$$= \frac{1}{Q} \sum_{i=0}^{R+Q-h} [F(R+Q-h-i;\lambda qL)\, p(i;\lambda pR_A) - F(R-h-i;\lambda qL)\, p(i;\lambda pR_A)]$$

$$= \frac{1}{Q}\left\{ \sum_{i=0}^{R+Q-h} p(i;\lambda pR_A)\, F(R+Q-h-i;\lambda qL) - \sum_{i=0}^{R+h} p(i;\lambda pR_A)\, F(R-h-i;\lambda qL) \right\}$$

Since $H(t) = Y(t) - X(t)$ it is easy to find the expected value of stock on hand:

$$E[H(t)] = E[Y(t)] - E[X(t)]$$

Richards showed that the expected value of net inventory for an <R,Q> policy is $R + \frac{Q+1}{2} - m$ where m is the mean number of failures lost to the system in a procurement lead time. Thus

$$E[H(t)] = R + \frac{Q+1}{2} - \lambda qL - \lambda pR_A$$

$$= R + \frac{Q+1}{2} - \lambda(pR_A + qL)$$

Now inventory position fluctuates between R+1 and R+Q, i.e.,

$$R+1 \leq Z(t) + Y(t) \leq R+Q$$

Assume the system starts with Y(0) = R + Q and thus Z(0) = 0. Then when Y(t) drops to R, Z(t) becomes Q. If this order does not come in by the time Y(t) = R - Q, then Z(t) becomes 2Q and so forth. The table below shows this relationship:

Y(t)	Z(t)
R + Q	0
⋮	⋮
R + 1	0
R	Q
⋮	⋮
R - Q + 1	Q
R - Q	2Q
⋮	⋮
R - 2Q + 1	2Q
⋮	⋮
R - (n-1)Q	nQ
⋮	⋮
R-(nQ) + 1	nQ

Then $Z(t) = nQ$ if and only if $R-nQ+1 \leq Y(t) \leq R-(n-1)Q \qquad n=1,2,\ldots$

$$P[Z(t)=nQ] = P[R-nQ+1 \leq Y(t) \leq R-nQ+Q]$$
$$= F_{Y(t)}(R-nQ+Q) - F_{Y(t)}(R-nQ)$$

where $F_{Y(t)}(x) = \sum_{y=0}^{x} P[Y(t)=y]$

Note that if Q=1, i.e. reorder every time an item is lost to the system, this reduces to

$$P[Z(t)=n] = F_{Y(t)}(R-n+1) - F_{Y(t)}(R-n)$$
$$= P[Y(t)=R-n+1]$$
$$= \frac{1}{Q}[\overline{F}(n;\lambda qL) - \overline{F}(n+1;\lambda qL)]$$
$$= \frac{(\lambda qL)^n e^{-\lambda qL}}{n!}$$

That is, when Q=1, the number on order at any time t, like the number in repair, is Poisson distributed as expected by Palm's Theorem.

Now $\quad I(t) = Y(t) + Z(t)$

$$E[I(t)] = E[Y(t)] + E[Z(t)]$$

$$r + \frac{Q+1}{2} = r + \frac{Q+1}{2} - \lambda qL + E[Z(t)]$$

or $\quad E[Z(t)] = \lambda qL$

Expressions have now been developed for X(t), Y(t), Z(t) and H(t) and the system is thus completely described. One measure of effectiveness that is almost always calculated for inventory models is the expected number of backorders at an arbitrary point in time. It is, of course

$$\sum_{b=1}^{\infty} b\ p[H(t)=-b]$$

where b = the number of backorders. Under previous definition the quantity on hand is considered a negative value when backorders exist, thus $H(t) = -b$ denotes b backorders on the records.

Another well-known measure of effectiveness is the expected number of backorders per unit time which is also given by the expected number of failures minus the expected number of fills. The expected number of failures is just λ, the rate of the Poisson failure process. Now a fill can occur only if $H(t)$, stock on hand, is greater than zero.

$$H(t) > 0 \Rightarrow Y(t) - X(t) > 0$$

$$\Rightarrow Y(t) > X(t)$$

Now $Y(t)$ is never greater than $R + Q$. If $Y(t) > X(t)$, but $X(t) > 0$ (always), then $Y(t) > 0$. So

$$p[\text{fill}] = p[Y(t) > X(t)] \quad \text{for} \quad 0 < Y(t) \leq R+Q$$

Therefore:

$$p[\text{fill}] = \sum_{y=1}^{R+Q} p[Y(t) > X(t) | Y(t)=y]\ p[Y(t)=y]$$

$$p[Y(t) > X(t) | Y(t)=y] = p[X(t) < y]$$

$$= \sum_{x=0}^{y-1} \frac{(\lambda pR_A)^x e^{-\lambda pR_A}}{x!}$$

$$= F(y-1;\lambda pR_A)$$

Then

$$p[\text{fill}] = \sum_{y=1}^{R+Q} F(y-1;\lambda pR_A)\ p[Y(t)=y]$$

$$= \sum_{y=1}^{R} p[Y(t)=y]\ F(y-1;\lambda pR_A) + \sum_{y=R+1}^{R+Q} p[Y(t)=y]\ F(y-1;\lambda pR_A)$$

$$= \sum_{y=1}^{R} \frac{1}{Q} [F(R+Q-y;\lambda qL) - F(R-y;\lambda qL)]\, F(y-1;\lambda pR_A)$$

$$+ \sum_{y=R+1}^{R+Q} \frac{1}{Q} F(R+Q-y;\lambda qL)\, F(y-1;\lambda pR_A)$$

$$= \frac{1}{Q} \left\{ \sum_{y=1}^{R+Q} F(R+Q-y;\lambda qL)\, F(y-1;\lambda pR_A) - \sum_{y=1}^{R} F(R-y;\lambda qL)\, F(y-1;\lambda pR_A) \right\}$$

Then the expected number of backorders is $\lambda - \lambda \cdot p[\text{fill}]$

$$= \lambda - \frac{\lambda}{Q} \left\{ \sum_{y=1}^{R+Q} F(R+Q-y;\lambda qL)\, F(y-1;\lambda pR_A) - \sum_{y=1}^{R} F(R-y;\lambda qL)\, F(y-1;\lambda pR_A) \right\} .$$

C. AN <S-1,S> POLICY WITH STOCHASTIC REPAIR AND PROCUREMENT LEAD TIMES

The <S-1,S> policy has often been suggested as especially appropriate for repairable items since the object of the policy is to maintain the total system stock at the level S, i.e. when an item fails and cannot be repaired another is immediately placed on order. Assume that there is no lag between a non-repairable failure and the placing of an order. Then for this model

$$X(t) + H(t) + Z(t) = S \text{ for all } t > 0$$

The number in repair at any time t, $X(t)$, and the number on order at any time t, $Z(t)$ are modeled as infinite server queues, with arrival rates λp and λq respectively, where $q = 1-p$. In this model both repair time and procurement lead times are random variables with means R_A and L respectively. Thus, Palm's queuing theorem is applicable to both $X(t)$ and $Z(t)$ giving

$$P[X(t)=x] = \frac{(\lambda pR_A)^x e^{-\lambda pR_A}}{x!} \qquad x=0,1,2,\ldots$$

$$P[Z(t)=z] = \frac{(\lambda qL)^z e^{-\lambda qL}}{z!} \qquad z=0,1,2,\ldots$$

i.e. both X(t) and Z(t) are Poisson distributed.

Now note that since $X(t) + H(t) + Z(t) = S$

$$Y(t) + Z(t) = S$$

then $\quad p[Y(t)=y] = p[Z(t)=S-y] \qquad y$ an integer, $-\infty \leq y \leq S$

$$= \frac{(\lambda qL)^{S-y} e^{-\lambda qL}}{(S-y)!}$$

Also $\quad p[H(t)=h] = p[Y(t)-X(t)=h] \qquad h$ an integer, $-\infty \leq y \leq S$

$$= \sum_{y=h}^{S} p[Y(t)=y, X(t)=y-h]$$

$$= \sum_{y=h}^{S} p[Y(t)=y]\, p[X(t)=y-h]$$

$$p[H(t)=h] = \sum_{y=h}^{S} \frac{(\lambda qL)^{S-y} e^{-\lambda qL}}{(S-y)!} \frac{(\lambda pR_A)^{y-h} e^{-\lambda pR_A}}{(y-h)!}$$

Feeney and Sherbrooke [Ref. 4], using a slightly different model, with an <S-1,S> policy, suggested the following three measures of supply performance (redefined for the present model):

(1) R(S), called the ready rate, is the steady state probability that S or less units are in resupply. For the present model "in resupply" means in repair plus on order.

$$R(S) = p[X(t)+Z(t) \leq S]$$

$$= \sum_{w=0}^{S} p[X(t)+Z(t)=w] \qquad \text{since both } X(t) \text{ and } Z(t) \text{ are non-negative.}$$

$$= \sum_{w=0}^{S} \left[\sum_{x=0}^{w} p(X(t)=x, Z(t)=w-x) \right]$$

$$= \sum_{w=0}^{S} \sum_{x=0}^{w} p[X(t)=x] \, p[Z(t)=w-x]$$

$$R(S) = \sum_{w=0}^{S} \sum_{x=0}^{w} \frac{(\lambda pR_A)^x e^{-\lambda pR_A}}{x!} \frac{(\lambda qL)^{w-x} e^{-\lambda qL}}{(w-x)!}$$

(2) F(S), called fills, is the expected number of demands per time period that can be filled immediately from stock on hand. As in the last section this is just the failure rate times the probability of a fill, i.e. λ p[fill]

$$p[\text{fill}] = p[H(t) > 0]$$

$$= p[X(t) < Y(t)]$$

$$= \sum_{y=1}^{S} p[X(t) < y | Y(t)=y] \, p[Y(t)=y]$$

$$= \sum_{y=1}^{S} p[X(t) < y] \, p[Y(t)=y]$$

$$= \sum_{y=1}^{S} F(y-1; \lambda pR_A) \frac{(\lambda qL)^{S-y} e^{-\lambda qL}}{(S-y)!}$$

$$F(S) = \lambda \sum_{y=1}^{S} F(y-1; \lambda pR_A) \frac{(\lambda qL)^{S-y} e^{-\lambda qL}}{(S-y)!}$$

Now, it is well known that for a Poisson demand distribution the <R,Q> and <s,S> policies are the same, with R + Q = S and Q = S-s. In this case, with an <S-1,S> policy R + Q = S, R = S - 1, and Q = 1. In the previous section it was shown that for the <R,Q> model,

$$p[\text{fill}] = \frac{1}{Q}\left\{\sum_{y=1}^{R+Q} F(R+Q-y;\lambda qL)\, F(y-1;\lambda pR_A) - \sum_{y=1}^{R} F(R-y;\lambda qL)\, F(y-1;\lambda pR_A)\right\}$$

Substituting the values for R+Q, R, and Q:

$$p[\text{fill}] = \sum_{y=1}^{S} F(S-y;\lambda qL)\, F(y-1;\lambda pR_A) - \sum_{y=1}^{S-1} F(S-y-1;\lambda qL)\, F(y-1;\lambda pR_A)$$

$$= \sum_{y=1}^{S-1} [F(S-y) - F(S-y-1);\lambda qL]\, F(y-1;\lambda pR_A) + F(0;\lambda qL)\, F(y-1;\lambda pR_A)$$

$$= \sum_{y=1}^{S-1} p(S-y;\lambda qL)\, F(y-1;\lambda pR_A) + F(0;\lambda qL)\, F(y-1;\lambda pR_A)$$

Now $F(0;\lambda qL) = p(0;\lambda qL) = p(S-S;\lambda qL)$.

Thus

$$p[\text{fill}] = \sum_{y=1}^{S} p(S-y;\lambda qL)\, F(y-1;\lambda pR_A)$$

which agrees with the expression given above for the <S-1,S> policy.

The expected number of backorders in a unit time is, again,

$$\lambda - \lambda \cdot p[\text{fill}]$$

$$= \lambda - \lambda \sum_{y=1}^{S} F(y-1;\lambda pR_A)\, p(S-y;\lambda qL)$$

(3) A third measure of supply performance considered is S(S), the number of "units in service," defined as the expected number of units in routine resupply at a random point in time. Routine resupply is further defined as the number of units in resupply minus the number of units backordered. Again "in resupply" is taken to mean in repair plus on order. We note that if $X(t) + Z(t) = r \leq S$, then $H(t) \geq 0$, i.e., there are no backorders, and the number of units in routine resupply is r.

On the other hand if $X(t) + Z(t) = r \geq S$, $H(t) < 0$, i.e., there are backorders, and the number of units in routine resupply is S.

Thus

$$S(S) = \sum_{r=0}^{S} r\, P[X(t)+Z(t)=r] + \sum_{r=S}^{\infty} S\, P[X(t)+Z(t)=r]$$

$$= \sum_{r=0}^{S} r \left\{ \sum_{w=0}^{r} P[X(t)=w]\; p[Z(t)=r-w] \right\} + \sum_{r=S}^{\infty} S \left\{ \sum_{w=0}^{r} P[X(t)=w]\; p[Z(t)=r-w] \right\}$$

$$S(S) = \sum_{r=0}^{S} r \left\{ \sum_{w=0}^{r} \frac{(\lambda pR_A)^w e^{-\lambda pR_A}}{w!} \; \frac{(\lambda qL)^{r-w} e^{-\lambda qL}}{(r-w)!} \right\}$$

$$+ S \sum_{r=S}^{\infty} \left\{ \sum_{w=0}^{r} \frac{(\lambda pR_A)^w e^{-\lambda pR_A}}{w!} \; \frac{(\lambda qL)^{r-w} e^{-\lambda qL}}{(r-w)!} \right\}$$

IV. ANALYSIS OF THE MODEL UNDER COMPOUND POISSON DEMAND

The model in the last section assumed that failures were a Poisson process of rate λ, i.e. the number of failures in a time interval of length T has a Poisson probability mass function with parameter λT. This implied failures occurred one at a time, and thus each requisition for replacement was for exactly one item.

In practice, the Poisson model may be quite reasonable for major repairable units, i.e. items of high cost and/or large, complex structure. However, there are smaller not so expensive items that are also economically repairable and that may experience failures in batches, i.e. more than one at a time. One good example might be the printed circuit "cards" found in many modern computers, radars, and other electronic equipment. These cards are generally easily removed and installed, and under conditions of peak load, current surges, etc., several may fail at the same time. Generally, the only damage may be to the resistors, capacitors, transistors, etc. which are soldered onto the printed circuit, while the circuit card itself is undamaged and repair is feasible.

These types of items lead to the consideration of failure processes in which the number of failures in a given time period is a compound Poisson process $\{W(t), t>0\}$. This process can also be represented by

$$W(t) = \sum_{n=1}^{V(t)} U_n$$

where $\{V(t), t>0\}$ is a Poisson process which is the number of customer requisitions received and $\{U_n, n=1,2,...\}$ is a family of independent identically distributed random variables where U_n is the number of items requested or the n^{th} requisition. It is necessary that $\{V(t), t>0\}$ and

U_n are assumed to be independent and for this model that means that the number of items per requisition is independent of which requisition it is, which is an entirely reasonable assumption. It is noted that the simple Poisson process is just a special case of a compound Poisson process, where the probability $\{U_n=1, \text{for any } n\} = 1$. The compound Poisson is an extremely rich family of distributions. Various seemingly quite different processes can be described merely by the proper choice of a compounding distribution.

Assume that W(t) the number of items which fail in a unit time interval is a compound Poisson process with rate λ and that U_n is distributed as a random variable U, with probability mass function f_u and characteristic function ϕ_U. Then it has been shown [Parzen, p. 130] that the characteristic function of W(t) is

$$\phi_{W(t)}(x) = e^{\lambda t(\phi_U(x)-1)}$$

Referring to the repairable item inventory model of this paper, assume that <u>all</u> failure items in a particular batch are either repairable or all are non-repairable and result in a procurement. Further, let the probability that all items in a batch are repairable be r. Then the number of items to enter repair from the n^{th} batch, R_n, is

U_n with probability r

0 with probability 1-r

That is $\quad \text{prob } [R_n = u] = \text{prob } [U_n = u]\, r \qquad u=1,2,3,\ldots$

$\text{prob } [R_n = 0] = 1-r$

The characteristic function of R_n is

$$\phi_{R_n} = E[e^{i\theta R_n}] = \sum_{j=0}^{\infty} e^{i\theta j}\, p[R_n=j]$$

$$= p[R_n=0] + \sum_{j=1}^{\infty} e^{i\phi j} \, p[R_n=j]$$

$$= 1-r + r \sum_{j=1}^{\infty} e^{i\phi j} \, p[U_n=j]$$

$$= 1-r + r\phi_{U_n}$$

$$\phi_{R_n} = 1 - r + r\phi_U$$

Then $N_1(t)$, the number of items entering repair in a time interval $(0,t]$, is

$$N_1(t) = \sum_{n=1}^{V(t)} R_n$$

and from above

$$\phi_{N_1(t)}(x) = e^{\lambda t(\phi_{R_n}(x)-1)}$$

$$= e^{\lambda t(1-r+r\phi_U-1)}$$

$$= e^{\lambda tr(\phi_U-1)}$$

which is just the same compound Poisson process as failures with rate λ replaced by rate λr. Likewise, $N_2(t)$, the number of items which are placed on order in an interval $(0,t]$, can be shown to be the same compound Poisson process with rate $\lambda(1-r)$.

Now, let the number of batches entering repair be $V_1(t)$ and the number of batches which must be replaced by procurements be $V_2(t)$, so that $V(t) = V_1(t) + V_2(t)$. The number of batches in the compound Poisson model corresponds to the number of items in the simple Poisson model. In Section II it was shown that the number of items entering repair, $N_1(t)$, and the number of items lost to the system, $N_2(t)$, were

independent random variables. In the compound Poisson process, this implies that $V_1(t)$ and $V_2(t)$ are independent. Let $N_1(t)$ be the number of failures in an interval $(s, s + t)$ which can be repaired, and let $N_2(t)$ be the number of failures in the same interval which are lost to the system. Then

$$N_1(t) = \sum_{n=1}^{V(t)} U_n X_1$$

$$N_2(t) = \sum_{n=1}^{V(t)} U_n X_2$$

where $X_1 = \begin{cases} 1 & \text{if the } n^{th} \text{ batch goes to repair} \\ 0 & \text{otherwise} \end{cases}$

$X_2 = \begin{cases} 0 & \text{if the } n^{th} \text{ batch goes to repair} \\ 1 & \text{otherwise} \end{cases}$

By assumption $\{U_n, n=1,2,\ldots\}$ is a family of independent identically distributed random variables, and a batch is either repairable or lost to the system. Thus $N_1(t)$ and $N_2(t)$ are sums of independent identically distributed random variables, and, furthermore, no U_n which is in the sum for $N_1(t)$ is also in the sum for $N_2(t)$ and vice versa. Therefore, $N_1(t)$ and $N_2(t)$ are independent. Further, $X(t)$, the number of items in repair at any time t is a function only of $N_1(t)$ and the repair queue parameters. Likewise, $Z(t)$, the number of items on order at any time t is a function only of $N_2(t)$, the procurement policy, and the procurement queue parameters. Thus $X(t)$ and $Z(t)$ are still independent for the compound Poisson case, assuming a batch goes completely to repair or is completely lost to the system.

Again defining $H(t)$ to be the number of items on hand at time t, and calling $Y(t) = X(t) + H(t)$ the net inventory at time t, the same analysis

as in Section II shows that $X(t)$ and $Y(t)$ are independent. Using the notation of that section

$$X(t) = C_1 + N_1(t) - D_1(t)$$

$$Y(t) = C_3 - N_2'(t) + D_2(t)$$

where $C_1 = X(0)$, $C_3 = Y(0)$, and $N_2'(t)$ is the number of items which have been ordered in the interval $(0,t]$. It has been shown that $N_1(t)$ and $N_2(t)$ are independent and it is assumed that $D_1(t)$ is a function only of $N_1(t)$, i.e., that the number of items repaired in an interval of length t is a function only of the number of items which enter the repair queue and the queue operation itself, and that $N_2'(t)$ and $D_2(t)$ are functions of $N_2(t)$. Thus $X(t)$ and $Y(t)$ are functions of independent random variables and are themselves independent random variables.

Assume that the inventory system is governed by an <S-1,S> policy whereby every failure that cannot be repaired is immediately placed on order. This, of course, may result in an order for more than one item under the assumption that an entire batch is either repairable or not. Assume further that all items which enter repair or which are placed on order at the same time have the same repair time or procurement lead time respectively. The effect of these assumptions is that items which fail in the same batch are either repairable and are repaired as a batch, or that those which are not repairable must be ordered and are received into stock as a batch. While these assumptions may seem unduly restrictive, in actual practice they are quite reasonable. Certainly items ordered at the same time are generally received at the same time. Further, repair often consists of a series of operations to be performed. It is

not unusual that the first operation is completed on all items in a batch before the second operation is started, etc., and upon completion of repair the items are returned to stock as a batch.

Let R_A be mean repair time and L be the mean procurement lead time. Let $f_u(x)$ be the "compounding" probability mass function, that is $f_u(x) = P\{U = x\}$.

Now Feeney and Sherbrooke have shown that under the above assumptions given a compound Poisson arrival density of rate λ into a queue, the steady state probability of x units in the queue is the same compound Poisson density with rate λT, where T is the mean service time in the queue. Since the rate of items entering repair is λr:

Prob {x units in repair at any time}

$$= \text{Prob}\{X(t) = x\}$$

$$= \sum_{i=0}^{x} \frac{(\lambda r R_A)^i e^{-\lambda r R_A}}{i!} f_u^{i*}(x)$$

where f_u^{i*} is the i-fold convolution of f_u.

Likewise, with the rate "into" procurement being $\lambda(1-r)$,

Prob{z units on order at any time}

$$= \text{Prob}\{Z(t) = z\}$$

$$= \sum_{i=0}^{z} \frac{[\lambda(1-r)L]^i e^{-\lambda(1-r)L}}{i!} f_u^{i*}(z)$$

Then assuming the <S-1,S> policy, where an item is ordered every time it is lost to the system, the inventory position, which is equal to on hand + in repair + on order - backorders, is always equal to S. In terms of the model

$$H(t) + X(t) + Z(t) = S$$

or $Y(t) + Z(t) = S$

Then $\text{Prob}\{Y(t)=y\}$

$$= \text{Prob}\{S-Z(t)=y\}$$

$$= \text{Prob}\{Z(t)=S-y\}$$

$$= \sum_{i=0}^{S-y} \frac{[\lambda(1-r)L]^i e^{-\lambda(1-r)L}}{i!} f_u^{i*}(S-y)$$

Furthermore, since $Y(t) = H(t) + X(t)$

$$H(t) = Y(t) - X(t) \qquad -\infty < H(t) \leq S$$

Then $\text{Prob}\{H(t)=h\}$

$$= \text{Prob}\{Y(t)-X(t)=h\}$$

$$= \sum_{y=0}^{S-h} \text{Prob}\{Y(t)=y+h, X(t)=y\}$$

$$= \sum_{y=0}^{S-h} \text{Prob}\{Y(t)=y+h\}\ \text{Prob}\{X(t)=y\}$$

and since $Y(t)$ and $X(t)$ are independent

$$= \sum_{y=0}^{S-h} \left[\sum_{i=0}^{S-y-h} \frac{[\lambda(1-r)L]^i e^{-\lambda(1-r)L}}{i!} f_u^{i*}(S-y-h) \sum_{i=0}^{y} \frac{(\lambda r R_A)^i e^{-\lambda r R_A}}{} f_u^{i*}(y) \right]$$

The system is now described for any time $t \geq 0$. Again, the measures of supply performance suggested by Feeney and Sherbrooke are ready rate, fills and units in service.

Ready rate, $R(S)$ is the steady state probability that S or less units are in repair or on order, i.e. $X(t) + Z(t) \leq S$.

$$R(S) = \text{Prob}\,[X(t) + Z(t) \leq S]$$

$$= \sum_{w=0}^{S} \text{Prob}\,[X(t) + Z(t) = w]$$

$$= \sum_{w=0}^{S} \sum_{x=0}^{w} \text{Prob}\,[X(t)=x, Z(t)=w-x]$$

$$\sum_{w=0}^{S} \sum_{x=0}^{w} \text{Prob}\,[X(t)=x]\ \text{Prob}\,[Z(t)=w-x]$$

$$= \sum_{w=0}^{S} \sum_{x=0}^{w} \left[\sum_{i=0}^{x} \frac{(\lambda r R_A)^i e^{-\lambda r R_A}}{i!} f_u^{i*}(x) \sum_{i=0}^{w-x} \frac{[\lambda(1-r)L]^i e^{-\lambda(1-r)L}}{i!} f_u^{i*}(w-x) \right]$$

Fills, F(S), is the expected number of demands per time period for an item that can be filled immediately from stock on hand. Feeney and Sherbrooke reasoned that, "There will always be one fill if S-1 or less units are in resupply [in repair plus on order] since each customer places an order for at least one unit. There will be a second fill if S-2 or less units are in resupply and the customer order is for two or more demands..."

Therefore, F(S) is the expected number of customers per time period, i.e. λ, times the expected number of fills per customer. As shown above, this can be represented as

$$
\begin{aligned}
F(S) = \lambda \Big\{ & (\text{probability a customer's demand is greater than } 0) \cdot \\
& (\text{probability } X(t) + Z(t) \leq S-1) \\
+ \ & (\text{probability a customer's demand is greater than } 1) \cdot \\
& (\text{probability } X(t) + Z(t) \leq S-2) \\
+ \ & \\
& \ldots\ldots \\
+ \ & (\text{probability a customer's damand is greater than } S-1) \cdot \\
& (\text{probability } X(t) + Z(t) \leq 0.) \Big\}
\end{aligned}
$$

$$
\begin{aligned}
= \lambda \Big\{ & 1 \cdot R(S-1) + [1-f_u(1)]R(S-2) + [1-f_u(1) - f_u(2)]R(S-3) + \\
& \ldots + [1-f_u(1)-f_u(2)-\ldots-f_u(S-1)]\ R(0) \Big\}
\end{aligned}
$$

This is the same formula given by Feeney and Sherbrooke, except the ready rate has been adjusted for the double "sources" of supply, i.e. repair and procurement. Then the expected number of backorders in a unit time is, with probability one, the expected number of demands less the expected number of fills, i.e.

$$E[\text{number of backorders}] = \lambda E[U] - F(S)$$

The other measure of supply performance is units in service, S(S), which is the expected number of units in routine resupply at a random point in time. There are r items in routine resupply if $X(t) + Z(t) = r \leq S$, and there are S items in routine resupply if $X(t) + Z(t) \geq S$. Thus

$$S(S) = \sum_{r=1}^{S} r \, \text{Prob}[X(t)+Z(t)=r] + \sum_{r=S+1}^{\infty} S \, \text{Prob}[X(t)+Z(t)=r]$$

$$= \sum_{r=1}^{S} r \sum_{w=0}^{r} \text{Prob}[X(t)=w] \, \text{Prob}[Z(t)=r-w]$$

$$+ \sum_{r=S+1}^{\infty} S \sum_{w=0}^{r} \text{Prob}[X(t)=w] \, \text{Prob}[Z(t)=r-w]$$

$$= \sum_{r=1}^{S} r \sum_{w=0}^{r} \left[\sum_{i=0}^{w} \frac{(\lambda r R_A)^i e^{-\lambda r R_A}}{i!} f_u^{i*}(w) \sum_{i=0}^{r-w} \frac{[\lambda(1-r)L]^i e^{-\lambda(1-r)L}}{i!} f_u^{i*}(r-w) \right.$$

$$+ S \sum_{r=S+1}^{\infty} \sum_{w=0}^{r} \left[\sum_{i=0}^{w} \frac{(\lambda r R_A)^i e^{-\lambda r R_A}}{i!} f_u^{i*}(w) \cdot \right.$$

$$\left. \sum_{i=0}^{r-w} \frac{[\lambda(1-r)L]^i e^{-\lambda(1-r)L}}{i!} f_u^{i*}(r-w) \right]$$

V. CONCLUSIONS

The results developed in this paper are relatively straightforward extensions of models and results found by Hadley and Whitten and by Feeney and Sherbrooke. The model is a fairly good approximation of reality. The assumption of Poisson or compound Poisson demand processes has found much favor in inventory theory. The steady state probability of the number of items in repair, in stock, or on order, have been found, under various inventory policies. From these probabilities, formulas for several measures of supply performance have been derived. Other measures can be developed in a similar manner. Thus, given the steady state system probabilities and the derived measures of effectiveness, inventory managers can now attempt to optimize the system under various criteria.

Further analysis to improve the model may also be possible; for instance, the probability of a fill given a delay time τ in which to make the fill would be a useful result. It may also be possible to relax the assumption that under compound Poisson demand, failure batches either go entirely to repair, or are lost to the system. It would seem more natural that each item of each batch may go into repair or may be lost to the system.

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13. ABSTRACT

A model is constructed of a repairable item inventory system where on-hand stocks are resupplied both by repair and procurement. The basic properties of the model are analyzed. A Poisson failure process is assumed, and steady state probabilities are found for operation of the model under <R,Q> and <S-1,S> inventory policies. Further, a compound Poisson failure process under an <S-1,S> inventory policy is also analyzed. Measures of supply performance, such as expected number of backorders per unit time, are derived for the system under the various policies.

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Repairable Item						
Stochastic Process						

DD FORM 1 NOV 65 1473 (BACK)

S/N 0101-807-6821

Security Classification

A-31409